Self-Excited Oscillations in the Sliding of Two Elastic Bodies With a Constant Friction Coefficient

Professor George G. Adams
Mechanical Engineering Department
Northeastern University
Boston, MA, USA, adams@coe.neu.edu
Self-Excited Oscillation

Instability due to a perturbation in velocity.

\[ m\ddot{x} + kx = \mu mg \]

\[ \mu = \mu_0[1 - \alpha(v_{rel} - v_0)] \]

\[ m\ddot{x} - (\alpha\mu_0 mg)\dot{x} + kx = \mu_0 mg \]

Just as if:

\[ m\ddot{x} + c\dot{x} + kx = F_0 \]

\[ c < 0 \quad \text{Negative Damping} \]
Violin String

$$v_{rel}^A < v_{rel}^B \Rightarrow \mu^A > \mu^B$$

$$|W^A| > |W^B|$$
Rayleigh Wave

\[ u_j(x_1,x_2,t) = A_j e^{s_j x_2} e^{i k (x_1 - c_R t)} \]
Stoneley Wave

Wave Decays Exponentially with Distance from Interface
Generalized Rayleigh Wave (Slip Wave) (Achenbach and Epstein, JEM, 1967)

Unbonded, Frictionless

\[ u_j(x_1,x_2,t) = A_j e^{-s_j x_2} e^{i k (x_1 - c_{GR} t)} \]

\[ u_j(x_1,x_2,t) = B_j e^{r_j x_2} e^{i k (x_1 - c_{GR} t)} \]

Decays Exponentially with Distance from Interface
Frictional Slip Wave
Adams (JAM, 1995)

Unbonded, Frictional Interface

\[ u_j(x_1, x_2, t) = A_j e^{-s_j x_2} e^{\Lambda t} e^{ik(x_1 - c_{FS}t)} \]
\[ u_j(x_1, x_2, t) = B_j e^{r_j x_2} e^{\Lambda t} e^{ik(x_1 - c_{FS}t)} \]

Increases with time

Decays Exponentially with Distance from Interface

\( \Lambda > 0 \)
Depending on the phase relation between $C$ and $D'$ more/less energy is dissipated at the interface than supplied remotely.
Frictional Slip-Waves

- Effect of friction is to destabilize slip waves.
- Steady-state sliding is unstable for a wide range of material combinations and friction coefficients.
- This behavior occurs even for infinitesimal sliding speeds.
- The magnitude of $\Lambda^R$ scales as the inverse wavelength.
\[ \begin{align*}
\nu_0 &= \sigma_0, \quad \nu = \nu' = 0.25 \\
\kappa &= \frac{c'_s}{c_s} \\
-\zeta &= \frac{\Lambda^R}{\Lambda^I}
\end{align*} \]
Sliding continues with increasing slip-wave amplitude until:

- Amplitude of relative velocity due to the slip wave equals the sliding velocity
  OR
- Amplitude of slip-wave pressure equals the applied pressure
  OR
- Nonlinearities or material failure
Stick-Slip Waves

- Stick-and-slip waves can propagate along a sliding interface.
- These waves are confined to the interface.
Stick-and-Slip Waves

- Stick-and-slip waves exist for a wide range material combinations and friction coefficients.

- The *observed* (measured, $\mu^*$) friction coefficient differs from the *interface* (local, $\mu$) friction coefficient.

- Specifically

$$\mu^* = \mu \left[ 1 - \beta \frac{(v_0 / c_s')}{(p^* / G') (\pi - \alpha)} \right]$$

- These stick-slip waves cause steady motion of the bodies away from the interface.
Interface Friction Coefficient is the ratio of shear stress to normal stress at the interface.

Observed Friction Coefficient is the ratio of shear force to normal force away from the interface.

These studies show that these friction coefficients are different.

Interface friction can be constant, but observed friction will decrease as the sliding velocity increases.
Layered Elastic Bodies

- Dry frictional sliding of layered bodies is dynamically unstable for a wide range of material combinations.

- The dynamic normal and shearing stresses along the bonded interface can be greater than the steady values.

- Dynamic stresses are apt to play a role in delamination.
Radiation of Energy Away From the Sliding Interface

- Elastic waves can be radiated away from the sliding interface.
- This phenomenon provides a mechanism for energy dissipation within the bulk of the material.
- Observed friction is less than interface friction.
- Observed friction decreases with speed even though interface friction is speed-independent.
Ultrasonic Propulsion Using Incident Square Waves

- Elastic dilatational square waves, incident upon the sliding interface, can reduce friction to zero or negative values.
- This phenomenon may lead to a new type of ultrasonic propulsion.
- Reflection consists of a pair of body waves (dilatational and shear) in each material.
Summary

- The steady sliding of two elastic bodies is dynamically unstable for a wide range of material combinations, friction coefficients, and low/high sliding speeds.

- This instability can cause stick-slip waves to propagate along the interface resulting in an observed friction coefficient which is different than the interface friction coefficient.